Equating Powers

Whenever you see 4 you should think 2^2 .

Whenever you see 8 you should think 2^3 .

Whenever you see 27 you should think 3^3 .

Whenever you see 64 you should think 4^3 or 8^2 or 2^6 .

Etcetera...

Also, pro tip; when you see 1 you should think 2^0 or 3^0 or 5^0 or ...

All of these problems reduce to

$$k^{\text{fish}} = k^{\text{dog}} \implies \text{fish} = \text{dog}.$$

Questions

1. Solve
$$2^{x+1} = 4^x$$
.

$$x = 1$$

14. Solve
$$8 \times \frac{2^{2-x}}{4^{3-2x}} = \frac{8^x}{4^{2+x}}$$
.

$$x=-\frac{3}{2}$$

2. Solve
$$8^{3x} = 2^{x-2}$$
.

$$x = -\frac{1}{4}$$

15. Solve
$$1 = 8^2 \times 4^{x-1} \times 2^{x+1}$$
.

$$x = -\frac{5}{3}$$

3. Solve
$$3^{2x-1} = 9^{4x}$$
.

$$x = -\frac{1}{6}$$

16. Solve
$$\frac{7^x}{49^{6-x}} = \frac{343^{1-x}}{7^{2x-3}}$$
.

$$\frac{9}{4}$$

4. Solve
$$27^{3-x} = 81^{2x}$$
.

17. Solve
$$8^{ax} = 4^{bx+1}$$
.

$$x = \frac{2}{3a - 2b}$$

5. Solve
$$2 \times 2^{2x+1} = 8^{x-1}$$
.
6. Solve $5^x \times 25^3 = 5^{2x-1}$.

$$x = 5$$

18. Solve
$$27^{ax+b} = 3^{cx-a}$$
.

$$x = \frac{a+3b}{c-3a}$$

19. Solve
$$9^{ax} \times 27^x \times 3^{ax} = 1$$
.

$$x = 0$$

7. Solve
$$4^x \times 8^{x-1} = 2 \times 4^{3x+1}$$
.

8. Solve $8 \times 2^{x-1} = 4^{2x-1}$.

20. Solve
$$\frac{5^{ax-2}}{25^{b-x}} = 125^{cx+d}$$
.

$$x = \frac{3d + 2 + 2b}{a + 2 - 3c}$$

9. Solve
$$16 \times 8^{2x+1} = 2 \times 16^x$$
.

$$\frac{1}{2} = -3$$
 21. Solve $\frac{2^{ax}}{2^{3-bx}} = \frac{4^{4+cx}}{8^{k-x}}$.

10. Solve
$$32^x = \frac{8^x}{2^{x+1}}$$
.

$$x = -\frac{1}{3}$$

Only do the following if you've studied solving quadratics by factorisation.

11. Solve
$$\frac{2^{x+1}}{4} = \frac{4^{x+3}}{8^x}$$
.

$$x = \frac{7}{2}$$

22. Solve
$$4^{x+2} = 2^{x^2+5}$$
.

$$x = 1$$
 (repeated)

12. Solve
$$\frac{9^x}{27^{x+1}} = \frac{3^x}{81^{1-x}}$$
.

23. Solve
$$3^{x^2+2} = 27^x$$
.

24. Solve $5^{2x^2} = 25^{2x+3}$.

13. Solve
$$\frac{5^{-2x+1}}{25^{3-x}} = \frac{125^x}{5^{x-4}}$$
.

$$x = -\frac{9}{2}$$

25. Solve
$$(x^2 + 5x + 5)^{x^2 + 11x + 30} = 1$$
.

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